Dirac Delta Function
Summary of Different Forms

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	∞ **Volume** (No boundary conditions)	**Finite Volume** (Boundary conditions)
Volume = 4D spacetime *V*	$\delta^{(4)}(x-y)=\frac{1}{(2\pi)^4}\int e^{-i(x-y)p}d^4p$ Continuous momenta p ∞ or finite p volume See bottom LH or bottom RH block Continuous x or discrete x_n	$\delta^{(4)}(x-y)=\frac{1}{V}\sum_{n=-\infty}^{\infty}e^{-i(x-y)p_n}$ Discrete momenta p_n ∞ or finite p volume See bottom LH or bottom RH block Continuous x or discrete x_n
Volume = 4D momentum space V_p	$\delta^{(4)}(p'-p)=\frac{1}{(2\pi)^4}\int e^{-i(p'-p)x}d^4x$ Continuous spacetime x ∞ or finite x volume See top LH or top RH blocks Continuous p or discrete p_n	$\delta^{(4)}(p'-p)=\frac{1}{V_p}\sum_{n=-\infty}^{\infty}e^{-i(p'-p)x_n}$ Discrete spacetime events x_n ∞ or finite x volume See top LH or top RH blocks Continuous p or discrete p_n

Note: Since integrals and sums are over all negative and positive values, the relations are equally valid with $-i$ replaced by i in the exponentials above.